Bus service time estimation model for a curbside bus stop

Bomin Bian[1]

Institute of Systems Engineering, College of Management and Economics, Tianjin University, China

Ning Zhu[1,*]

Institute of Systems Engineering, College of Management and Economics, Tianjin University, China

Shuai Ling[1]

Institute of Systems Engineering, College of Management and Economics, Tianjin University, China

Shoufeng Ma[1]

Institute of Systems Engineering, College of Management and Economics, Tianjin University, China

*Corresponding author

Email addresses: bmbian@tju.edu.cn (Bomin Bian), zhuning@tju.edu.cn (Ning Zhu), lingshuai@tju.edu.cn (Shuai Ling), sfma@tju.edu.cn (Shoufeng Ma)

Abstract

The bus service time at bus stop areas occupies a large proportion of the total on-road bus operational time. Curbside bus stops are very common in urban transit systems, and the occurrence of bus queues forming at the entry and departure area of bus stop is quite frequent. To estimate the service time at a curbside bus stop, a compound Poisson service time estimation model (CPSTM) is proposed. The CPSTM considers the interactions among arriving buses and number of boarding and alighting passengers. Realistic observational data are acquired for a representative bus stop. Four different scenarios are presented to estimate the total expected service time. The service time estimation of each bus line is obtained via the CPSTM, and the effectiveness of the proposed CPSTM is demonstrated. The results show that the employment of real-time data is not required for accurate service time estimation.

List of keywords

1. Bus service time
2. Probabilistic model
3. Multiple bus lines
4. Interaction among buses
5. Bus stop

1. Introduction

With the severe congestion in urban transportation system, calls are increasing for a switch from private cars to public transit. Bus on-road travel time is an important measure of the performance of bus systems and, in general, this has two main components: the vehicle running time and service time at bus stops (Levinson (1983); Rajbhandari et al. (2003)). Bus stops are a major access point to the transit system and service time represents a significant proportion of the total bus on-road travel time (Lin and Wilson (1992); Lin and Bertini (2010)). It is fairly common in any country for a large number of bus lines to converge on a single bus stop. Particularly in high demand areas, curbside bus stops are frequent, despite the limited road space there. In addition, due to variations in on-road travel time and the difficulty of controlling a scheduled headway, the phenomenon of queuing is commonplace and often delays the service time at a bus stop. It is necessary, therefore, to propose a method for estimating service time in view of this queuing. Research into service time at bus stops is beneficial in several transit applications. Service time estimation could help in the estimation and prediction of bus travel time (Furth and Muller (2007)), and arrival and departure time forecasts of intelligent transportation systems could become more precise if service time at bus stops is modeled (Shalaby and Farhan (2004); Yu et al. (2011)). Service time estimation are vital to daily operations, such as bus scheduling (Petersen et al. (2012); Ceder (2011)) and headway control (Sun and Hickman (2008); Delgado et al. (2012)). Moreover, bus service time estimation plays an vital role in transit network design (Cipriani et al. (2011);Bagloee and Ceder (2011); Yu et al. (2012); Wu et al. (2015); Zhang et al. (2014); Yao et al. (2014); Amiripour et al. (2014); Nayeem et al. (2014)) and transit assignment analysis (Szeto et al. (2011); Hamdouch et al. (2011);Leurent et al. (2014)).

A number of factors have been shown to impact the amount of time buses spend at the bus stop areas, denoted as the bus dwell time. These factors are the number of boarding and alighting passengers, the number of berths of the bus stop, fare collection system, bus types, etc. One of the well-studied factors is that of the number of boarding and alighting passengers. Various regression models have been developed to describe the relationship between the number of passengers and the bus dwell time. Linear regression and natural logarithm models have been respectively presented by Levinson (1983) and Guenthner and Sinha (1983) to estimate bus dwell time. Linear and nonlinear estimation of crowding effects have been examined (Lin and Wilson (1992)). These models show the importance of the number of passengers on the dwell time estimation.

In addition to the number of boarding and alighting passengers, the effects of standees, vehicle types, fare collection systems, etc. have also been considered in the estimation of the bus dwell time. For example, the effects of standees and the time of the day on the bus dwell time were investigated (Rajbhandari et al. (2003)). Jaiswal et al. (2010) studied the influence of platform walking on the bus dwell time at a bus rapid transit station and developed a linear regression model that not only considered the number of boarding and alighting passengers but also considered the bus lost time caused by waiting the first boarding passenger walking to the bus. Guenthner and Hamat (1988); Levine and Torng (1994), Milkovits (2008) and Tirachini (2013) estimated the dwell time using some secondary factors such as fare collection methods and bus types, which were employed as variables to obtain a regression model. Meng and Qu (2013) provided a new effective mathematical approach for estimation of the bus dwell time, and presented a probabilistic model to describe the dwell time incurred when a bus remains in the bus bay in anticipation of a suitable time gap to depart. Most of the aforementioned studies focused on the dwell time in the bus bay area.

Intuitively, there are two processes going on during bus service time at stops (Fernandez (2010)). One is passengers' boarding and alighting process. The other is the buses arrival process which can potentially form a queue of buses at the stop area. Due to different numbers and functions of bus doors, the service time of passengers' boarding and alighting varies (Fernandez and Planzer (2002)). The other part of the service procedure is the time taken for buses to enter and leave the service area. Bus travel time is highly variable in a congested environment. Normally, there is a timetable for buses leaving the terminus, but, headway is difficult to control at each stop. Therefore, a congested traffic environment leads to a bunching phenomenon and a great deal of queuing, with buses tending to arrive in groups and cause delays for others entering the area of the stop (Strathman et al. (2000); Tirachini and Hensher (2011); Tirachini (2013)).

In general, there are five main reasons for the queuing phenomenon at bus stop areas. (i) Passengers waiting at the bus stop areas normally form a disordered crowd, which increases the delay, as shown in Fig. 1(a). This part of service time is the passengers' boarding and alighting time. (ii) Numerous bus lines (perhaps 5-20 lines, as shown in Fig. 1(b)) may operate through a single bus stop area, so, it is inevitable that a number of buses will enter the curbside bus stop area simultaneously, as shown in Fig. 1(c). (iii) The number of service berths of the curbside bus stop is limited. A single bus stop cannot simultaneously accommodate multiple buses, as shown in Fig. 1(d). (iv) Irregular headway and headway variations increase the possibility of bunching. (v) Complicated traffic conditions (e.g. needing to wait for a suitable gap to merge back into the main traffic stream (Meng and Qu (2013)) may increase service time. Observations indicate that only two or three buses can provide service simultaneously. Curbside bus stops are very common all over the world. Recently, there have been limited studies focusing on bus queue related problems. A berth assignment model is proposed that considers a bus stop shared among multiple bus lines (Tan et al. (2014)). A Markov chain based model is proposed to estimate the maximum number of arriving buses at a given bus stop (Gu et al. (2014)).

As to the analytical tool for bus stop capacity analysis, a good example is the Highway Capacity Manual formula (TRB (2013)). This connects the number of berths and the stop capacity (Vuchic (2005)) but its simplicity acts as a disadvantage as it can not precisely capture the queuing situation. On the other hand, because of the complexity of interaction among passengers, buses and bus stop layout (Gibson et al. (1989)), it is difficult to invent a closed-form expression to reflect these realistic phenomena. Therefore, simulation tools are widely used (Fernandez and Planzer (2002); Fernandez (2010); Gibson et al. (1989); Tyler et al. (2002)Rexfelt et al. (2014)). It is necessary to develop a new model to estimate bus service time at curbside bus stops that considers the commonly experienced queuing phenomenon.

[Figure 1 about here.]

The remainder of this paper is organized as follows. Section 2 provides a description of the bus stop area considered herein and the data collection and processing employed. In section 3, we present a new model that is based on a compound Poisson process to describe the service time at a curbside bus stop. A numerical experiment and calculation results and analysis are given in section 4 and 5, respectively. Section 6 concludes the paper.

2. Description of bus stops and data

2.1. Bus stop description

The curbside bus stop is the most common type of bus stop all over the world. The structure of a typical curbside bus stop is shown in Fig. 2. The area inside the blue dashed lines is the service area. This study specifies the number of berths as 2 because this is the most commonly observed berth number based on our observations. This indicates that buses residing outside the service area must wait before the service can be provided.

Moreover, a "no overtaking rule" is imposed as an assumption. This assumes that no bus in a queue can be overtaken by buses further back in the queue. The reason for this assumption is that (i) the space of the service area is insufficient to allow such a maneuver; and (ii) the maneuver would affect traffic flow in the other lanes and may cause accidents.

[Figure 2 about here.]

The process of bus service at stops can be described as the following steps:

(1) The entering bus is positioned in the lane that contains the curbside bus stop.

(2) The speed of the bus is reduced while approaching the bus stop service area and the queuing phenomenon may occur due to possibly happened service of forward buses at the same time.

(3) Upon entering the service area, the front and rear doors are opened for passengers boarding and alighting.

(4) Passengers in the bus adjust their positions, and the front and rear doors are closed.

(5) The bus attempts to leave the bus stop area and enter traffic, where delays may occur because of the possibly happened service of forward buses simultaneously.

(6) The bus merges into the main traffic flow.

2.2. Data collection and processing

To estimate the bus dwell time under conditions of queuing at the bus stop area, an actual curbside bus stop was subjected to observation and evaluation. A total of nine different bus lines operate through the observed bus stop located on the Anshanxi Road in the Nankai district, Tianjin, China. A digital camera was used to record the bus dwell times occurring during service periods. The duration of the record was from 15:00 to 16:20 every weekday in April 2014 which was non-peak hour. A total of 282 records were obtained with an average of 32 records for each bus line. All these records provide the authentic service time information.

Collected data are analyzed manually by three master students and cross validation are conducted to guarantee that all input data are consistent. Input data are extracted from the video based on the event sequence illustrated in Fig. 3. Several sample records are as following(Table 2):

[Table 1 about here.]

For all bus lines, totally 895 arrival records are analyzed. There are peak and non-peak hour data for each bus line. 18 groups of arrival data are used to justify the Poisson process assumption(McClave and Sincich (January 6, 2012)). 17 of total 18 groups passed the test. 12 out of 18 groups of data for inter-arrival times series passed exponential distribution. In addition, assumption of Poisson distribution has been widely employed(Danas (1980); Kohler (1991); Ge (2006); Gu and Cassidy (2013); Tirachini (2013)). Thus, we made the assumption of Poisson distribution.

3. Model description

3.1. Basic model and compound Poisson service time model (CPSTM)

In general, the buses that service in Tianjin usually have two doors. The front door is only used for boarding and the rear door only is for alighting. The bus design is fairly common all over the world. The service time estimation model based on the European experiences, which was proposed by Pretty and Russell (1988), is treated as the basic model in the present study. This model considers only the numbers of boarding and alighting passengers and the dead time.

$$T = C + \max\{\sum_{h=1}^{m} a_h, \sum_{q=1}^{n} b_q\}, \tag{1}$$

where:

T is the bus dwell time at the bus stop;
a_h is the consumed time of each passenger h for boarding;
b_q is the consumed time of each passenger q for alighting;
m is the number of boarding passengers;
n is the number of alighting passengers;
C is the dead time for opening and closing doors.

To ease the description of the notation system, we also provide a notation table of CPSTM in A.

Typically, there are two types of time delay that may occur at a curbside bus stop. In step (2) described above, a bus can enter the service area only if there is sufficient space. Otherwise, the bus must wait. The waiting time at the point of entry is the first type of time delay. The second type of delay may occur at step (5) with a similar logic. These two types of delays are partially caused by limited number of berths. At the same time, two other conditions have to be satisfied that are (i) forward berths are also occupied and (ii) the no overtaking rule is applied. Cases where the first bus is still boarding and/or alighting passengers while the second bus waits was often observed. Based on the above observations, the service time formulation is revised as follows:

$$T_s = T_d + T_m, \tag{2}$$

where:

$$T_d = C + \max\{\sum_{i=1}^{m} a_h, \sum_{j=1}^{n} b_q\} + t_{we} + t_{wl} = T + t_{we} + t_{wl}, \tag{3}$$

$$T_m = t_e + t_l. \tag{4}$$

Here, the definition of a_h, b_q, m, n and C are the same with equation (1)

T_s is the total service time at the bus stop;

T_d is the dwell time in and/or out of the bus stop(including the waiting time for entering and/or leaving the bus stop area);

T_m is the time wherein that buses move in and out of the bus stop;

t_{we} is the time wherein buses wait to enter the bus stop;

t_{wl} is the time wherein buses wait to leave the bus stop;

t_e is the time wherein buses enter the bus stop;

t_l is the time wherein buses leave the bus stop.

Moreover, the symbol "+" used as a superscript represents a forward bus. For example, T_s^+ represents the total service time of the forward bus at a given bus stop. In addition, in the case of a queue, T_s^{++} indicates the total service time of the bus in front of the described forward bus at the same bus stop. The variable t_{gap} is denoted as the time difference between the start point of t_{we}^+ and the start point of t_{we} of the current bus, as shown in Fig. 3. All the variables are non-negative.

[Figure 3 about here.]

3.2. Poisson process and compound Poisson process

Let $N(t)$ donates the occurrence number of any event A until time t. If $\{N(t), t \geq 0\}$ satisfies the following conditions:

(1) $N(0) = 0$;

(2) $N(t)$ is an independent increment process;

(3) For $N(t)$, the following two equations hold:

$$\begin{cases} P\{N(t+h) - N(t) = 1\} = \lambda h + o(h) \\ P\{N(t+h) - N(t) \geq 2\} = o(h), \end{cases} \tag{5}$$

then the stochastic process $\{N(t), t \geq 0\}$ is a Poisson process with constant intensity λ. During any time period $(0, t]$, if $N(t)$ is a Poisson distribution of constant intensity λ, then

$$P\{N(t+s) - N(s) = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!}. \tag{6}$$

It is assumed that $\{N(t), t \geq 0\}$ is a Poisson process and has a constant λ as a parameter. If $\{Y_k, k = 1, 2, \cdots\}$ is a set of independently and identically distributed random variables, and $X(t)$ is defined as:

$$X(t) = \sum_{k=1}^{N(t)} Y_k, t \geq 0, \tag{7}$$

we obtain that $\{X(t), t \geq 0\}$ is a compound Poisson process(Gallager (2013)).

In our context, $N(t)$ is the number of boarding passengers during time period $(0,t]$. As such, $Y_k(k = 1, 2, \cdots, N(t))$ denotes the boarding time of the $k-$th passenger. For equation (7), $X(t)$ is the total boarding time of all passengers.

$N(t)$ is a Poisson process with parameter λ and $X(t)$ is a compound Poisson process, then for any $t \ and \ s \in [0,\infty), s < t$,

$$E(N(t) - N(s)) = D(N(t) - N(s)) = \lambda(t - s) \tag{8}$$

and

$$E[X(t)] = \lambda t E[Y_1], \tag{9}$$

$$D[X(t)] = \lambda t E[Y_1^2] = \lambda t(D[Y_1] + (E[Y_1])^2). \tag{10}$$

Let $N_1(t)$ represents the number of buses that arrived at the curbside bus stop until time t. $T_n(n \geq 1)$ represents the time interval between the arrival time of the $(n-1)-$th bus and the arrival time of the $n-$th bus. $\{T_n, n \geq 1\}$ is the time interval series associated with $N_1(t)$. The stochastic variable $T_n(n = 1, 2, \cdots)$ is independently and identically and is subject to the exponential distribution with parameter λ. The mean value of T_n is $1/\lambda$.

3.3. Probabilistic model for bus service time in bus stop area

To study the queuing phenomenon, we must set a proper number of berths for the curbside bus stop in the CPSTM. The number of berths of a curbside bus stop mainly depends on its location as well as nearby traffic conditions. Based on our observations, stops with the number of berths of two are very common, whereas stops with the number of berths of three were only observed near the central business district. In this paper, we assume that the number of berths of service area is two. The results and conclusions can be extended to the value of three. As shown in Fig. 4(a), the service area is divided into berth-1 and berth-2, and each berth can accommodate a single bus. Therefore, a bus can encounter four different scenarios before moving into the service area. The occurrence of each scenario is associated with a probability. The four scenarios are stated as follows.

Scenario A: the service area is empty, as illustrated in Fig. 4(a). The bus can freely enter the service area and stop at berth-1. The bus can leave the service area immediately after service is completed.

Scenario B: the service area is full and the bus must wait in the entry area, as illustrated in Fig. 4(b). There are two sub-scenarios. One is that the berth-1 bus completes its service earlier than the berth-2 bus. Thus, the berth-1 bus leaves the bus stop area. The current bus must wait until the berth-2 bus leaves. The other sub-scenario is that the berth-2 bus first completes its service. However, the berth-2 bus must wait until the berth-1 bus leaves owing to the no overtaking rule. After the berth-1 bus completes its service, both buses can leave, and the current bus can enter the service area and stop at berth-1. Therefore, in scenario B, the current bus must wait until both front buses exit the service area, although, after service, the current bus can exit without waiting.

Scenario C: a single bus resides in the service area in berth-2, as illustrated in Fig. 4(c). This case is similar to scenario B, and the current bus again must wait to enter and stop at berth-1. The bus can exit the service area immediately after its service is completed.

Scenario D: a single bus resides in the service area in berth-1, as illustrated in Fig. 4(d). In this case, the current bus can stop at berth-2 directly without waiting. However, there are two scenarios when the bus prepares to leave. If the berth-1 bus is still servicing, then the current bus must wait because of the no overtaking rule. Otherwise, it can leave directly.

[Figure 4 about here.]

Of the four scenarios, the probability of each scenario was calculated. We assume J is the set of bus lines that utilize the curbside bus stop. For each $j \in J$, the probability of a scenario's occurrence is as follows.

Scenario A:

$$P_A^j = P\{t_{gap}^j \geq T_s^+\}, \tag{11}$$

Scenario B:

$$P_B^j = P\{t_{gap}^{+j} < T_s^{++}\} \times P\{t_{gap}^j < T_s^+\}, \tag{12}$$

Scenario C:

$$P_C^j = P\{t_{gap}^{+j} \geq T_s^{++}\} \times P\{t_{gap}^j < T_s^+\} \times p^*, \tag{13}$$

Scenario D:

$$P_D^j = P\{t_{gap}^{+j} \geq T_s^{++}\} \times P\{t_{gap}^j < T_s^+\} \times (1 - p^*). \tag{14}$$

Here, t_{gap}^j is the t_{gap} of the current bus line j and t_{gap}^{+j} is the t_{gap}^+ of the current bus line j. For scenario C and D, only one bus is in the service area. p^* is defined as the probability of the single bus is in berth-2 and $1 - p^*$ is the probability of a single bus at berth-1.

In these four probability models, only the current bus line is considered. The combined probability $P_C^j + P_D^j = P\{t_{gap}^{+j} \geq T_s^{++}\} \times P\{t_{gap}^j < T_s^+\}$ represents the condition where only a single bus is servicing at the bus stop.

Moreover,

$$P_C^j + P_D^j = P\{t_{gap}^{+j} \geq T_s^{++}\} \times P\{t_{gap}^j < T_s^+\} = \frac{P_D^j}{1 - p^*}. \tag{15}$$

Because a bus must stop at berth-2 for scenario D, we have

$$\sum_j P_D^j = p^*. \tag{16}$$

Then

$$\sum_j (P_C^j + P_D^j) = \frac{\sum_j P_D^j}{1 - P^*} = \frac{p^*}{1 - p^*}. \tag{17}$$

Because $\sum_j (P_C^j + P_D^j)$ can be determined from collected data, p^* can be obtained.

For any $j \in J$, we assume the number of coming j buses during time period $(0, t]$ is $N_j(t)$, and that $\{N_j(t), t \geq 0\}$ is a Poisson process with parameter λ_j. The series $\{t_{gap}^j\}$ and $\{t_{gap}^{+j}\}$ are subject to the exponential distribution with parameter λ_j. The mean value of t_{gap} and t_{gap}^+ are both $1/\lambda_j$. The probabilities of the different scenarios can then be calculated by the exponential distribution.

3.4. Description of the bus service time for different scenarios

In addition to the probability of each scenario, T_s is different for different scenarios. We assume that the length of a typical bus is L. The processes of a bus accelerating to leave and decelerating to enter can be seen as uniform variable motion (we assume that the acceleration and deceleration are constant), and the speeds are given as v_l and v_e respectively. Meanwhile, the boarding passenger time and alighting passenger time are both compound poisson processes with parameters μ_a^j and μ_b^j respectively, and y_a and y_b represent the respective mean value time values of alighting and boarding passengers. The values v_l, v_e, y_a, y_b are constant for all $j \in J$. For any $j \in J$, based on the four scenarios described before, T_s^j can be further computed as follows.

Scenario A:

$$t_{we}^j = t_{wl}^j = 0, t_e^j = \frac{L}{v_e}, t_l^j = \frac{L}{v_l}, \tag{18}$$

then

$$T_{sA}^j = C + \max\{\mu_a^j t_{gap}^j y_a, \mu_b^j t_{gap}^j y_b\} + \frac{L}{v_e} + \frac{L}{v_l}. \tag{19}$$

Scenario B:

$$t^j_{wl}=0, t^j_e=\frac{2L}{v_e}, t^j_l=\frac{L}{v_l}, \tag{20}$$

$$\begin{aligned} t^j_{we} &= t^+_{we}+t^+_e+T^++t^+_{wl}-t^j_{gap}=(t^+_{we}+t^+_e+T^++t^+_{wl}+t^+_l)-t^+_l-t_{gap} \\ &= T^+_s-\frac{L}{v_l}-t_{gap}, \end{aligned} \tag{21}$$

then

$$T^j_{sB}=C+\max\{\mu^j_a t^j_{gap}y_a, \mu^j_b t^j_{gap}y_b\}+T^+_s+\frac{2L}{v_e}-t_{gap}. \tag{22}$$

Scenario C:

$$t^j_{wl}=0, t^j_e=\frac{2L}{v_e}, t^j_l=\frac{L}{v_l}, \tag{23}$$

$$\begin{aligned} t^j_{we} &= t^+_{we}+t^+_e+T^++t^+_{wl}-t^j_{gap}=(t^+_{we}+t^+_e+T^++t^+_{wl}+t^+_l)-t^+_l-t_{gap} \\ &= T^+_s-\frac{L}{v_l}-t_{gap}, \end{aligned} \tag{24}$$

then

$$T^j_{sC}=C+\max\{\mu^j_a t^j_{gap}y_a, \mu^j_b t^j_{gap}y_b\}+T^+_s+\frac{2L}{v_e}-t_{gap}. \tag{25}$$

Scenario D:

$$t^j_{we}=0, t^j_e=\frac{L}{v_e}, \tag{26}$$

$$t^j_{wl}=\begin{cases} 0 & t^j_{we}+t^j_e+T+t_{gap}\geq t^+_{we}+t^+_e+T^++t^+_{wl} \\ t^+_{we}+t^+_e+T^++t^+_{wl}-t^j_{we}-t^j_e-T-t^j_{gap} & t^j_{we}+t^j_e+T+t_{gap}< t^+_{we}+t^+_e+T^++t^+_{wl} \end{cases} \tag{27}$$

$$t^+_{we}+t^+_e+T^++t^+_{wl}-t^j_{we}-t^j_e-T-t_{gap}=T^+_s-\frac{L}{v_e}-\frac{L}{v_l}-T-t_{gap}, \tag{28}$$

accordingly,

$$t^j_l=\begin{cases} \frac{L}{v_l} & t^j_{wl}=0 \\ \frac{2L}{v_l} & t^j_{wl}\neq 0, \end{cases} \tag{29}$$

then

$$T^j_{sD}=C+\max\{\mu^j_a t^j_{gap}y_a, \mu^j_b t^j_{gap}y_b\}+\frac{L}{v_e}+t^j_{wl}+t^j_l. \tag{30}$$

T^j_{sB} and T^j_{sC} are defined according to equation (22) and (25), but their meanings are different depending on the scenario. In scenario B, the t^+_{wl} is nonnegative because the front bus in berth-2 may wait to leave; and, in scenario C, t^+_{wl} is 0 because only a single bus resides in the service area.

To estimate the service time of a specified bus line at a curbside bus stop, the mathematical expectation is calculated as follows:

$$T^j_s=\sum_i p^j_i T^j_{si}, \qquad i\in\{A,B,C,D\}, j\in J. \tag{31}$$

3.5. Summary of CPSTM and basic model

All parameters in CPSTM may be extracted from historical data, therefore, it can be regarded as an offline model. Most of these parameters are constant, such as C, y_a, y_b, L, v_e, v_l. Others are critical in obtaining good estimation performance, such as μ_a, μ_b, λ_j, t_{gap}, t^+_{gap}. μ_a, μ_b and λ_j should be updated in terms of different time intervals, say peak hour and non-peak hour.

A weakness of the basic model, is that it requires the number of boarding and alighting passengers, which is not easy to obtain in reality. In CPSTM, the number of passengers is not needed; instead, the rate of passenger arrival is used and obtainable from historical data, for example the transit fare system. The other weakness of the basic model is that it does not explicitly consider queueing which has a significant impact on bus service time.

4. Service time calculation

4.1. Calculation of the bus arrival rate

The curbside bus stop of interests contains nine bus lines. These 9 bus lines are No. 45, No. 50, No. 633, No. 662, No. 678, No. 842, No. 851, No. 859 and No. 879. The line set of this bus stop is then denoted as $J = \{45, 50, 633, 662, 678, 842, 851, 859, 879\}$. There are nine associated bus arrival rates λ_j that also serve as the parameters used for calculating the time interval series $\{t^j_{gap}\}$ and $\{t^{+j}_{gap}\}$. Moreover, we define λ_0 as the total bus arrival rate of the curbside bus stop and $\{t^0_{gap}\}$ is the corresponding time interval series. λ_j is calculated according to the arrival record of bus line j while λ_0 is calculated by the arrival record of all nine bus lines together. The results are presented in Table 3. The obtained arrival rate can be used for the computation of average passengers' waiting time, the prediction of bus arrival time and the computer simulation for generating bus.

[Table 2 about here.]

4.1.1. Probability of different scenarios

To calculate the mathematical expectation of T^j_s for each $j \in J$, p^j_i and T^j_{si}, $i \in \{A, B, C, D\}$ should be computed first, which is given in equation (31). P^j_i is calculated according to the following equations.

$$P^j_A = P\{t^j_{gap} \geq T^+_s\} = 1 - \int_0^{T^+_s} \lambda_j e^{-\lambda_j t} dt. \tag{32}$$

$$P^j_B = P\{t^{+j}_{gap} < T^{++}_s\} \times P\{t^j_{gap} < T^+_s\} = \int_0^{T^{++}_s} \lambda_j e^{-\lambda_j t} dt \times \int_0^{T^+_s} \lambda_j e^{-\lambda_j t} dt. \tag{33}$$

$$P^j_C = P\{t^{+j}_{gap} \geq T^{++}_s\} \times P\{t^j_{gap} < T^+_s\} \times P^* = (1 - \int_0^{T^{++}_s} \lambda_j e^{-\lambda_j t} dt) \times \int_0^{T^+_s} \lambda_j e^{-\lambda_j t} dt \times p^*. \tag{34}$$

$$P^j_D = P\{t^+_{gap} \geq T^{++}_s\} \times P\{t_{gap} < T^+_s\} \times (1 - p^*) = (1 - \int_0^{T^{++}_s} \lambda_j e^{-\lambda_j t} dt) \times \int_0^{T^+_s} \lambda_j e^{-\lambda_j t} dt \times (1 - p^*). \tag{35}$$

Here, T^+_s and T^{++}_s are two unknowns, which are also our estimation targets. They are obtained from historical data.

We define $\overline{T^0_s}$ as the mean service time of all buses that pass through the curbside bus stop, $\overline{T^j_s}$ ($j \in J$) as the mean service time of bus line j and n_j is the occurrence number of bus line j. Let $N = \sum\limits_{j \in J} n_j$, then

$$\overline{T^0_s} = \sum_{j \in J} \frac{n_j \overline{T^j_s}}{N}, \tag{36}$$

and

$$T_s^{++} = T_s^+ + t_{gap}^+ - t_{gap} = T_s^+ + t_{gap}^0. \tag{37}$$

The real data of t_{gap}^0 is replaced by the mathematical expectation $E[t_{gap}^0]$ as follows:

$$T_s^{++} = T_s^+ + E[t_{gap}^0] = T_s^+ + \frac{1}{\lambda_0}. \tag{38}$$

4.1.2. Bus service time for different scenarios

As for the bus service time for different scenarios, some parameters should be preprocessed. The deadtime C, bus length L, bus speeds v_l and v_e, and per passenger aligHing/boarding times y_a and y_b are all constant and calculated based on historical data. The values of all these constants are listed in Table 4. The time interval t_{gap} and t_{gap}^j are both replaced by the mathematical expectation $E[t_{gap}^j]$ since they are all time interval of current bus j. However, for some scenarios, the $E[t_{gap}^j]$ is greater than T_s^+, and thus, causes some negative parameters such as t_{we}^j for scenario B. Therefore, nonnegative constrains are added to avoid negative parameters.

Scenario A: For scenario A, there are no nonnegative constraints problem.

Scenario B:

$$t_{we}^j = t_{we}^+ + t_e^+ + T^+ + t_{wl}^+ - t_{gap} = max(0, T_s^+ - \frac{L}{v_l} - E[t_{gap}^j]). \tag{39}$$

Scenario C:

$$t_{we}^j = t_{we}^+ + t_e^+ + T^+ + t_{wl}^+ - t_{gap} = max(0, T_s^+ - \frac{L}{v_l} - E[t_{gap}^j]). \tag{40}$$

Scenario D:

$$t_{wl}^j = \begin{cases} 0 & t_{we}^j + t_e^j + T + t_{gap} \geq t_{we}^+ + t_e^+ + T^+ + t_{wl}^+ \\ max(0, T_s^+ - \frac{2L}{v} - T - E[t_{gap}^j]) & t_{we}^j + t_e^j + T + t_{gap} < t_{we}^+ + t_e^+ + T^+ + t_{wl}^+. \end{cases} \tag{41}$$

x

[Table 3 about here.]

4.1.3. Calculation of T_s^j

As described in Section 4.1.1, we use $\overline{T_s^0}$, which is initially obtained from historical data, to replace $T_s^+(=\overline{T_s^0})$ and $T_s^{++}(=\overline{T_s^0}+\frac{1}{\lambda_0})$ for calculating the probabilities of different scenarios and then for calculating T_s^j. However, the T_s^j is our estimation target, and T_s^+ as well as T_s^{++} also depends on T_s^j. The estimation results may not be accurate. Therefore, an iterative algorithm for accurately computing $T_s^j(j \in J)$ is designed[1].

The process of the iterative algorithm is given as follows.

Step 1: set $k = 0$, $\overline{T_s^j}$ and $\overline{T_s^0}$ are calculated based on historical data; set $\overline{T_s^j} = T_{s,k}^j$ and $\overline{T_s^0} = T_{s,k}$.

Step 2: $T_{s,k}$ is used to replace T_s^+ and $T_{s,k} + \frac{1}{\lambda_0}$ is used to replace T_s^{++}; thus, $T_{s,k+1}^j$ and $T_{s,k+1}$ can be calculated accordingly.

Step 3: if $T_{s,k}^j = T_{s,k+1}^j$, stop; otherwise, set $k = k + 1$ and go to step 2.

The computational results are listed in Table 5.

[Table 4 about here.]

[Figure 5 about here.]

[1]That stable bus service time estimations can be obtained by iteration shall be demonstrated in B

The computation converges to a solution very quickly, as shown in Fig. 5. By jointly utilizing scenarios probability and service time of each scenario, a bus online schedule model and headway control strategy may be proposed based on this calculation.

5. Model evaluation, comparison and discussion

5.1. Model performance evaluation

To evaluate the performance of the CPSTM, three error measurements are computed: the mean absolute error, root mean squared error, and mean relative error. These three errors are defined as follows.

Mean absolute error (ε^j_{mean}):

$$\varepsilon^j_{mean} = \frac{1}{n_j}\sum_{i=1}^{n_j} \mid T^j_s - T^j_{(i,s)} \mid . \tag{42}$$

Root mean squared error (μ^j_{mean}):

$$\mu^j_{mean} = \sqrt{\frac{1}{n_j}\sum_{i=1}^{n_j}(T^j_s - T^j_{(i,s)})^2}. \tag{43}$$

Mean relative error (ε^j_{rmean}):

$$\varepsilon^j_{rmean} = \frac{1}{n_j}\sum_{i=1}^{n_j} \frac{\mid T^j_s - T^j_{(i,s)} \mid}{T^j_{(i,s)}}. \tag{44}$$

Here:
T^j_s is the estimated service time of bus line j;
$T^j_{(i,s)}$ is the observed service time of the ith bus of line j.
The error measurements of the CPSTM are listed in Table 6.

5.2. Performance comparison of CPSTM and basic model

To compare the effectiveness of the basic model and our proposed CPSTM, error measurements for the basic model are also computed. In the calculation process of the CPSTM, the parameters C, λ_j, μ^j_a, μ^j_b, y_a and y_b are computed from historical data. The values for L, v_e and v_L are obtained from observations. In the basic model, the values of $C + \max\{\sum_{i=1}^m a_h, \sum_{j=1}^n b_q\}$ are from field. As to T_m which is the time spent moving in and out of the bus stop. It is not considered in basic model. The reason is that, in basic model, the dwell time is counted between the time of opening doors after stopping and closing them after last passenger getting on. The error measurements of the basic model are listed in Table 7. Moreover, the differences between error measurements of the two models are listed in Table 8.

[Table 5 about here.]

[Table 6 about here.]

[Table 7 about here.]

As shown in Table 6, the mean absolute errors and root mean squared errors of the CPSTM are no more than 5.82 s and 6.81 s, which are acceptable for actual applications. The mean relative errors of the CPSTM ranges from 15.1% to 32.1%. For the errors of the basic model, which are listed in Table 7, the mean absolute errors are no less than 7.44 s while the root mean squared errors are larger than 7.47 s and the mean relative errors are as large as 63.5%. Table 8 reflects the differences of error measurements of the basic model and the CPSTM. The three types of error measurements of the basic model are considerably larger than those of the CPSTM obviously except for route No. 851 in which the three errors of the basic model are only slightly larger than those of the CPSTM. The obvious differences reflect that the queuing time at the bus stop area can increase the estimation errors of the bus service time a great deal, and that ignoring the queuing phenomenon may reduce the effectiveness of the estimation. The results in Table 8 reflect that the accuracy as well as the credibility of the CPSTM is much better than the basic model.

5.3. Estimation of service time with observed t_{gap}, t^{+}_{gap} and t^{j}_{gap}

To further evaluate our model, the observed values of t_{gap}, t^{+}_{gap}, and t^{j}_{gap}, which were previously replaced by the mathematical expectation in $T_{si}, i \in A, B, C, D$ of the CPSTM , were employed to obtain new estimation results. Then, we compared the three error measurements, and the results are listed in Table 9.

[Table 8 about here.]

[Table 9 about here.]

Like Table 8, Table 10 is employed to compare the performances of the CPSTM and the CPSTM in conjunction with the observed data. In the CPSTM, the values of t_{gap} and t^{+}_{gap} are the critical data for estimating the queuing time, and t^{j}_{gap} is used to estimate the number of boarding and alighting passengers. From a theoretical perspective, these data are able to improve the estimation accuracy. The differences of errors shown in Table 10 are small, which indicates that the performances of the CPSTM with and without the observed data are nearly equivalent. Based on this observation, $E[t^{j}_{gap}]$ can be used for service time estimation and real-time data are not necessary. Based on the results shown in Table 8 and Table 10, the CPSTM is effective for estimating the bus service time for the curbside bus stops. The results show that a reasonable good estimation accuracy can be achieved without using any real-time data. This is quite useful for both the off-line and on-line bus schedule. In addition, the results can also be applied for headway control and bus holding strategy design.

5.4. Impact of overtaking

Based on our observed bus stop, the occurrence of overtaking is very rare. However, it happened occasionally for other bus stops. In this section, the impact of overtaking is examined. Overtaking means that the current bus overtakes forward bus either for entering the vacant berths or leaving the bus stop. To ease the analysis of overtaking, we assume that the coming bus overtakes servicing buses homogenously and with a fixed probability.

The overtaking analysis starts from the four scenarios shown in Fig. 6. For scenario A, since there is no buses in the stop area, the overtaking problem does not exist. This is named scenario E.

Scenario F: there are three possible scenarios deriving from scenario B assumed the occurrence of overtaking. (i) Berth-1 bus completes its service before berth-2 bus. Then, the coming bus overtakes berth-2 bus and stops at berth-1 position (see Fig. 6(a)). (ii) Berth-2 bus finishes its service before berth-1 and it overtakes berth-1 bus, then the current bus goes to berth-2. It becomes scenario D. (iii) Similar with (ii), after berth-2 bus overtakes berth-1 bus, the current bus overtakes berth-1 bus when it departs before berth-1 bus (see Fig. 6(b)).

Scenario G: based on scenario C, the coming bus overtakes berth-2 bus and provide service in berth-1 position. The current bus can discharge from the bus stop area after it completes its service (see Fig. 6(c)).

Scenario H: in terms of scenario D, there is a single bus residing the berth-1. The current bus can stop at berth-2 directly and overtakes the berth-1 bus if it finishes its service before berth-1 bus (see Fig. 6(d)).

[Figure 6 about here.]

To evaluate the impact of overtaking, a mathematical analysis is provided for the service time estimation of each extended scenarios and its probability of occurrence (see C). The service time of each scenario and its presence probability is seen as given. The time difference between no overtaking and overtaking with a specific probability is given in equation (62) and denoted as ΔT. There are two probabilistic variables for depicting the overtaking phenomena. One is the probability that the berth-1 bus finishes its service earlier than berth-2 bus in scenario B, which is denoted as $p^{'}$. The other probabilistic variable describes the homogeneous overtaking behavior which is denoted as p.

In terms of the equation (62), ΔT is a quadratic convex function of p and a linear function of $p^{'}$. In addition, ΔT is a monotonous function with respect to p on $[0, 1]$. Two groups of numerical experiments are conducted to see the impact of both $p^{'}$ and p on service time. $T_{sA}, T_{sB}, T_{sC}, T_{sD}, P_A, P_B, P_C, P_D$ and the minimum time waiting for either berth-1 bus or berth-2 bus to complete in scenario B are fixed. $P^{'} = 0.05$ and $P^{'} = 0.95$ are tested for ten different overtaking probabilities ranging from 0.1 to 1 with an interval of 0.1. The results are shown in Fig. 7.

[Figure 7 about here.]

The results indicates that (i) overtaking weakened the influence of queueing and decreases the service time at bus stop; (ii) the higher the overtaking probability, the more service time saving can be obtained; (iii) there is a slight increase in ΔT when the probability that berth-1 bus completes its service earlier than berth-2 bus is higher; (iv) if the probability of overtaking is 100%, the service time saving is the same no matter which bus finishes its service earlier in scenario B.

5.5. Impact of different stochastic arrival processes

In this section, different stochastic arrival rate of passengers are examined. The Monte Carlo method is employed to obtain service time estimation of different passengers' arrival distribution. Two other arrival processes are adopted, namely uniform distribution and normal distribution. The mathematical expectation of these two distribution are the same as the Poisson distribution. The variance of normal distribution is also the same as Poisson distribution. The variance of uniform distribution cannot be set, because it is dependent on its expectation.

[Table 10 about here.]

For each distribution, a hundred iterations of experiments were conducted. The mean values of different errors are provided in Table 11. By comparing the results of the Poisson arrival process and the normal process, it is clear that the Poisson process outperforms the normal process on most bus lines. When comparing the Poisson process and the uniform process, the former has a slight advantage. Besides, intuitively, the assumption of uniform arrival process is not reasonable. All in all, the Poisson arrival process is suitable for modeling passengers' arrival.

6. Conclusion

In this paper, we presented the new compound Poisson service time estimation model (CPSTM) to estimate bus service time at curbside bus stops, with particular consideration for the queuing phenomenon. Four different scenarios that may occur in curbside bus stops were established and the corresponding probability models were formulated based on the Poisson process.

We observed a representative curbside bus stop, and collected data every weekday in the month of April, resulting in the collection of 282 valid bus service events. By comparing the computational results of the basic model and the CPSTM, we analyzed the effectiveness of the CPSTM and found from Table 8 that the queuing phenomenon at the bus stop considerably increased the bus service time, requiring that it be taken into account. Moreover, the observed values of t_{gap}, t^{+}_{gap} and t^{j}_{gap} were also employed for the CPSTM computation, and the performances were nearly equivalent with and without the real-time data. The formulation of the CPSTM is therefore reasonable and effective.

Since CPSTM can be seen as an off-line model and offers an effective estimation, it facilitates a transportation agency's enhancement of its understanding of bus stop performance and may be connected to other transit modeling applications. First, CPSTM can be incorporated into a transit simulation tool because it provides a probabilistic description rather than deterministic results of bus service time. Probabilistic modeling of bus service time is more precise than deterministic model, due to its uncertain nature. Second, by applying a computer simulation, the relationship between the number of bus stop berths and capacity can be analyzed. Third, CPSTM may also be used in transit scheduling for either a single bus line or network-wide application. Last, service time at a bus stop area is an important component in the design of a transit network. CPSTM has the potential to be used in transit network design problems, particularly for those focusing on bus transfer.

As to the CPSTM itself, two possible directions present themselves for further exploration in the future. First, the number of berths of some curbside bus stops, such as stops in the central business district or large hub stops, can be expanded to 3, and the CPSTM accordingly revised. Second, passenger demand and traffic conditions vary during the day as well as the passenger arrival rate at the bus stop (Tirachini (2013)). The arrival rates λ and μ can be seen as functions of the time periods t. The compound Poisson process will be extended to inhomogeneous compound Poisson processes. These two time period functions can be formulated by regression models using related data.

7. Acknowledgements

This research was supported by the National Natural Science Foundation Council of China under projects 71301115, 71431005, 71271150 and 71101102, Specialized Research Fund for the Doctoral Program of Higher Education of China (SRFDP) under Grant No. 20130032120009 and Tianjin Social Science Planning Projects under Grant No. TJGL13-015.

A. Variables and parameters

Table 1: Variables and parameters

Variables or parameters	Definitions and source
T	bus dwell time which is used in basic model.
C	dead time for opening and closing doors which is obtained by observation.
a_h	the consumed time of each alighting passenger h which is from observation.
b_q	the consumed time of each boarding passenger q which is from observation.
m	the number of alighting passengers which is from observation.
n	the number of boarding passengers which is from observation.
T_s	the estimated service time of current bus which is the estimation target in CPSTM.
T_s^+ [a]	the estimated service time of the forward bus.
T_s^{++} [b]	the estimated service time of the front bus of the forward bus.
T_d	the dwell time in and/or out of the bus stop(including the waiting time for entering and/or leaving the bus stop area). It is a component in T_s.
T_m	the time wherein that buses move in and out of the bus stop.
t_{we}	the time wherein buses wait to enter the bus stop.
t_{we}^+	the time wherein the forward buses wait to enter the bus stop.
t_{we}^{++}	the time wherein the bus that is front of the forward bus wait to enter the bus stop.
t_{wl}	the time wherein buses wait to leave the bus stop.
t_{wl}^+	the time wherein the forward buses wait to leave the bus stop.

Continued on next page

Table 1 – *Continued from previous page*

Variables or parameters	Definitions and source
t_e	the time wherein buses enter the bus stop.
t_e^+	the time wherein the forward buses enter the bus stop.
t_l	the time wherein buses leave the bus stop.
t_l^+	the time wherein the forward buses leave the bus stop.
t_{gap}	the time difference between the start point of t_{we}^+ and t_{we}. It is obtained from observation.
t_{gap}^+	the time difference between the start point of t_{we}^{++} and t_{we}. It is obtained from observation.
t_{gap}^j [c]	the t_{gap} of the bus line j.
t_{gap}^{+j}	the time difference between the start point of t_{we}^{++} and t_{we} of bus line j. It is obtained from observation.
P_i^j	the probability of presence of scenario i for bus line j where $i \in \{A, B, C, D\}$
p^*	the probability of only one bus servicing in the stop area at berth-2.
L	the length of a typical bus.
v_l	the speed when buses are leaving the curbside bus stop.
v_e	the speed when buses are entering the curbside bus stop.
λ_j	the bus arrival rate of bus line j.
λ_0	the total bus arrival rate, $\lambda_0 = \sum \lambda_j$
μ_a^j	the arrival rate of alighting passengers of bus line j.
μ_b^j	the arrival rate of boarding passengers of bus line j.
y_a	mean value time of alighting passenger.
y_b	mean value time of boarding passenger.
T_s^j	the estimated service time of bus line j.
T_{si}^j	the total service time of scenario i of bus line j where $i \in \{A, B, C, D\}$.
$\overline{T_s^0}$	the mean value of estimated service time of all bus lines.
$\overline{T_s^j}$	the mean value of estimated service time of bus line j.
n_j	the number of observed buses of line j.
N	the number of total observed buses.
$T_{s,k}$	the total service time calculated by iteration k.
$T_{s,k}^j$	the total service time of bus line j calculated by iteration k.
$T_{(i,s)}^j$	the observed service time of the i-th bus of line j

[a] The sign "+" in the superscript position denotes a forward bus. If a variable is superscripted with "+", it means the associated value of its forward bus. For example, T_{we}^+ means the time that the immediate forward bus wait to enter the bus stop.

[b] The sign "++" in the superscript position denotes the bus in front of the forward bus. If a variable is superscripted with "++", it means the associated value of its front bus of the forward bus. For example, T_s^{++} means the total service time of the bus in front of the forward bus at the same bus stop.

[c] If a variable is superscripted with j , it means the associated value of bus line j. For example, t_{gap}^j is the t_{gap} of bus line j.

B. Proof of existence of stable solution by iteration

As shown in the iteration process, $T_{s,k}$ and $T_{s,k}^j$ will update according to the iteration times k until $T_{s,k}^j = T_{s,k+1}^j$, which indicates that the estimation of $T_{s,k}^j$ is stable (then $T_{s,k}$, according to equation (36), will be stable too).

For any $j \in J$, equation (31) can be expanded as follows.

$$
\begin{aligned}
T_s^j = \sum_i p_i^j T_{si}^j &= p_A^j T_{sA}^j + p_B^j T_{sB}^j + p_C^j T_{sC}^j + p_D^j T_{sD}^j \\
= (1 - \int_0^{T_s^+} \lambda_j e^{-\lambda_j t} dt) \cdot T_{sA}^j s &+ \int_0^{T_s^{++}} \lambda_j e^{-\lambda_j t} dt \cdot \int_0^{T_s^+} \lambda_j e^{-\lambda_j t} dt \cdot T_{sB}^j \\
&+ (1 - \int_0^{T_s^{++}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{T_s^+} \lambda_j e^{-\lambda_j t} dt \cdot p^* \cdot T_{sC}^j \\
&+ (1 - \int_0^{T_s^{++}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{T_s^+} \lambda_j e^{-\lambda_j t} dt \cdot (1 - p^*) \cdot T_{sD}^j.
\end{aligned}
$$

Let $T_s^+ = x$, then $T_s^{++} = x + \frac{1}{\lambda_0}$. Set:

$$
\begin{aligned}
f(x) = (1 - \int_0^{x} \lambda_j e^{-\lambda_j t} dt) \cdot T_{sA}^j s &+ \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot T_{sB}^j \\
&+ (1 - \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot p^* \cdot T_{sC}^j \\
&+ (1 - \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot (1 - p^*) \cdot T_{sD}^j.
\end{aligned}
$$

If $f(x) = x$ has solutions, then, according to the fixed point theorem, we can obtain a stable T_s^j by iteration.

Let

$$
\begin{aligned}
g(x) = f(x) - x = (1 - \int_0^{x} \lambda_j e^{-\lambda_j t} dt) \cdot T_{sA}^j s &+ \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot T_{sB}^j \\
&+ (1 - \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot p^* \cdot T_{sC}^j \\
&+ (1 - \int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt) \cdot \int_0^{x} \lambda_j e^{-\lambda_j t} dt \cdot (1 - p^*) \cdot T_{sD}^j - x,
\end{aligned}
$$

where $g(x)$ is a continuous function on $(-\infty, +\infty)$. It is apparent that the solutions of $g(x) = 0$ must distribute on $(0, +\infty)$ because x is the estimation of the bus service time, which must be positive. When $x \to 0$, $\int_0^x \lambda_j e^{-\lambda_j t} dt \to 0$, and $g(x) \to T_{sA}^j$, which is absolutely positive, and, when $x \to \infty$, both $\int_0^x \lambda_j e^{-\lambda_j t} dt$ and $\int_0^{x+\frac{1}{\lambda_0}} \lambda_j e^{-\lambda_j t} dt$ approach 1 such that $g(x) = T_{sB}^j - x = -\infty$. According to the intermediate value theorem, $g(x) = 0$ has solutions on $(0, +\infty)$. However, we may have more than one x that satisfies $g(x) = 0$. If that occurs, we can apply solutions that match the physical significance of x.

C. Calculating the probability and service time of scenarios in section 5.4

For scenarios E, F, G and H, the service time of each scenario is calculated as following:

Scenario E:

same as scenario A, then

$$
T_{sE}^j = T_{sA}^j = C + \max\{\mu_a^j t_{gap}^j y_a, \mu_b^j t_{gap}^j y_b\} + \frac{L}{v_e} + \frac{L}{v_l}. \tag{45}
$$

Scenario F:

similar to scenario B,

$$t_{wl}^j = 0, t_e^j = \frac{L}{v_e}, t_l^j = \frac{L}{v_l}, \tag{46}$$

$$t_{we}^j = \min\{T_s^{++} - \frac{L}{v_l} - t_{gap}^+, T_s^+ - \frac{L}{v_l} - t_{gap}\}, \tag{47}$$

then

$$\begin{aligned} T_{sF}^j &= C + \max\{\mu_a^j t_{gap}^j y_a, \mu_b^j t_{gap}^j y_b\} + \frac{L}{v_e} + \frac{L}{v_l} + \min\{T_s^{++} - \frac{L}{v_l} - t_{gap}^+, T_s^+ - \frac{L}{v_l} - t_{gap}\} \\ &= T_{sA}^j + \min\{T_s^{++} - \frac{L}{v_l} - t_{gap}^+, T_s^+ - \frac{L}{v_l} - t_{gap}\}. \end{aligned} \tag{48}$$

Scenario G:
same as scenario E, then

$$T_{sG}^j = T_{sE}^j = C + \max\{\mu_a^j t_{gap}^j y_a, \mu_b^j t_{gap}^j y_b\} + \frac{L}{v_e} + \frac{L}{v_l}. \tag{49}$$

Scenario H:
same as scenario E, then

$$T_{sH}^j = T_{sE}^j = C + \max\{\mu_a^j t_{gap}^j y_a, \mu_b^j t_{gap}^j y_b\} + \frac{L}{v_e} + \frac{L}{v_l}. \tag{50}$$

Fixed solutions can be obtained for T_{sE}^j, T_{sF}^j, T_{sG}^j and T_{sH}^j by iteration procedure proved in B.

In addition to service time of each scenario, the probability of occurrence of each each scenario should be calculated. It is assumed the probability of bus overtaking is p. The probabilities of the four scenarios A, B, C and D are P_A, P_B, P_C and P_D respectively. The probability of berth-1 bus finishes its service first in scenario B is $p^{'}$. Thus, the probability of berth-2 bus finishes its service first in scenario B is $1 - p^{'}$. $T_s^{'}$ represents estimated bus service time when overtaking is allowed and with a fixed probability. $T_{si}^{'}$, $i \in \{A, B, C, D\}$, represents the associated bus estimated service time of scenario i. Therefore, $T_s^{'} = \sum_i T_{si}^{'} P_i$. $\Delta T (= T_s - T_s^{'})$ is defined as the difference between T_s and $T_s^{'}$. Without loss of generality, we set

$$T_{sA} = a, T_{sB} = b, T_{sC} = c, T_{sD} = d, \tag{51}$$

so

$$T_{sE} = a, T_{sF} = a + m, T_{sG} = a, T_{sH} = a, \tag{52}$$

according to the calculations above, where $m = \min\{T_s^{++} - \frac{L}{v_l} - t_{gap}^+, T_s^+ - \frac{L}{v_l} - t_{gap}\} > 0$. Moreover, a, b, c, d, m satisfy follows:

$$a + m \le b, a + m \le c, a + m \le d, c \le b, d \le b, \tag{53}$$

since b and c contain time value of waiting to enter or exit, d contains time value of waiting to enter and exit,and m is the minimum of waiting time. Thus,

$$T_s = P_A T_{sA} + P_B T_{sB} + P_C T_{sC} + P_D T_{sD} = P_A \cdot a + P_B \cdot b + P_C \cdot c + P_D \cdot d, \tag{54}$$

$$T_s^{'} = P_A T_{sA}^{'} + P_B T_{sB}^{'} + P_C T_{sC}^{'} + P_D T_{sD}^{'}, \tag{55}$$

$$T_{sA}^{'} = T_{sE} = a, \tag{56}$$

$$\begin{aligned} T_{sB}^{'} &= p^{'} \cdot [(1-p) \cdot T_{sB} + p \cdot T_{sF}] + (1-p^{'}) \cdot \{p \cdot [p \cdot (T_{sF} + (1-p) \cdot (T_{sD} + m)] + (1-p) \cdot T_{sB}\} \\ &= p^{'}[(1-p) \cdot b + p \cdot (a+m)] + (1-p^{'})\{p \cdot [p \cdot (a+m) + (1-p) \cdot (d+m)] + (1-p) \cdot b\} \\ &= b + [(a-d) \cdot p^{'} + d + m - b] \cdot p + (1-p^{'}) \cdot (a-d) \cdot p^2. \end{aligned} \tag{57}$$

$$T_{sC}^{'} = (1-p) \cdot T_{sC} + p \cdot T_{sG} = (1-p) \cdot c + p \cdot a = c + (a-c) \cdot p, \tag{58}$$

$$T_{sD}^{'} = (1-p) \cdot T_{sD} + p \cdot T_{sH} = (1-p) \cdot d + p \cdot a = d + (a-d) \cdot p, \tag{59}$$

$$\begin{aligned} T_s^{'} =& P_A \cdot a + P_B \cdot b + P_B \cdot \{[(a-d) \cdot p^{'} + d + m - b] \cdot p + (1-p^{'}) \cdot (a-d) \cdot p^2\} \\ &+ P_C \cdot c + P_C \cdot (a-c) \cdot p + P_D \cdot d + P_D \cdot (a-d) \cdot p, \end{aligned} \tag{60}$$

$$\begin{aligned} \Delta T =& T_s - T_s^{'} \\ =& - P_B \cdot \{[(a-d) \cdot p^{'} + d + m - b] \cdot p + (1-p^{'}) \cdot (a-d) \cdot p^2\} \\ & - P_C \cdot (a-c) \cdot p - P_D \cdot (a-d) \cdot p \\ =& - P_B \cdot (1-p^{'}) \cdot (a-d) \cdot p^2 \\ & - \{P_B \cdot [(a-d) \cdot p^{'} + d + m - b] \cdot p + P_C \cdot (a-c) + P_D \cdot (a-d)\} \cdot p \end{aligned} \tag{61}$$

Noticed that P_A, P_B, P_C, P_D, p and $p^{'}$ are probabilities which belong to $[0,1]$ and $a < c, a < d, d + m \leq b$, then

$$\Delta T = -P_B \cdot (1-p^{'}) \cdot (a-d) \cdot p^2 - \{P_B \cdot [(a-d) \cdot p^{'} + d + m - b] \cdot p + P_C \cdot (a-c) + P_D \cdot (a-d)\} \cdot p > 0. \tag{62}$$

References

Amiripour, S.M., Ceder, A.A., Mohaymany, A.S., 2014. Designing large-scale bus network with seasonal variations of demand. Transportation Research Part C: Emerging Technologies 48, 322–338.

Bagloee, S.A., Ceder, A.A., 2011. Transit-network design methodology for actual-size road networks. Transportation Research Part B: Methodological 45(10), 1787–1804.

Ceder, A.A., 2011. Public-transport vehicle scheduling with multi vehicle type. Transportation Research Part C: Emerging Technologies 19(3), 485–497.

Cipriani, E., Gori, S., Petrelli, M., 2011. Transit network design: A procedure and an application to a large urban area. Transportation Research Part C: Emerging Technologies 20(1), 3–14.

Danas, A., 1980. Arrivals of passengers and buses at two london bus stops. Traffic Engineering and Control 21(10), 472–475.

Delgado, F., Munoz, J.C., Giesen, R., 2012. How much can holding and/or limiting boarding improve transit performance? Transportation Research Part B: Methodological 46(7), 1202–1217.

Fernandez, R., 2010. Modelling public transport stops by microscopic simulation. Transportation Research Part C: Emerging Technologies 18(6), 856–868.

Fernandez, R., Planzer, R., 2002. On the capacity of bus transit systems. Transport Reviews: A Transnational Transdisciplinary Journal 22(3), 267–293.

Furth, P.G., Muller, T.H.J., 2007. Part 4: Capacity and quality of service: Service reliability and hidden waiting time: Insights from automatic vehicle location data. Journal of the Transportation Research Board 1995(1), 79–87.

Gallager, R.G., 2013. Stochastic processes: theory for applications. Cambridge University Press.

Ge, H., 2006. Traffic impacts of bus stops in urban area and related optimization techniques. Ph.D. thesis. Southeast University, China.

Gibson, J., Baeza, I., Willumsen, L., 1989. Bus-stops, congestion and congested bus-stops. Traffic engineering and control 30(6), 291–302.

Gu, W., Cassidy, M.J., 2013. Maximizing bus discharge flows from multi-berth stops by regulating exit maneuvers. Transportation Research Part B: Methodological 56, 254–264.

Gu, W., Cassidy, M.J., Li, Y., 2014. Models of bus queueing at curbside stops. Transportation Science .

Guenthner, R.P., Hamat, K., 1988. Transit dwell time under complex fare structure. Journal of Transportation Engineering 113(3), 367–379.

Guenthner, R.P., Sinha, K.C., 1983. Modeling bus delays due to passenger boardings and alightings. Journal of Transportation Research Board 915, 7–13.

Hamdouch, Y., Ho, H., Sumalee, A., Wang, G., 2011. Schedule-based transit assignment model with vehicle capacity and seat availability. Transportation Research Part B: Methodological 45(10), 1805–1830.

Jaiswal, S., Bunker, J., Ferreira, L., 2010. Influence of platform walking on brt station bus dwell time estimation: Australian analysis. Journal of Transportation Engineering 136, 1173–1179.

Kohler, U., 1991. Capacity of transit lanes, in: In: Proceedings of the International Symposium on Highway Capacity, Karlsruhe, Germany.

Leurent, F., Chandakas, E., Poulhs, A., 2014. A traffic assignment model for passenger transit on a capacitated network: Bi-layer framework, line sub-models and large-scale application. Transportation Research Part C: Emerging Technologies 47, 3–27.

Levine, J.C., Torng, G.W., 1994. Dwell-time effects of low-floor bus design. Journal of Transportation Engineering 120(6), 914–929.

Levinson, H.S., 1983. Analyzing transit travel time performance. Transportation Research Record: Journal of the Transportation Research Board , 1–6.

Lin, T.m., Wilson, N.H.M., 1992. Dwell time relationships for light rail systems. Journal of Transportation Research Board 1361, 287–295.

Lin, W.H., Bertini, R.L., 2010. Modeling schedule recovery processes in transit operations for bus arrival time prediction. Journal of Advanced Transportation 38(3), 347–365.

McClave, J., Sincich, T., January 6, 2012. Statistics (12th Edition). Pearson.

Meng, Q., Qu, X., 2013. Bus dwell time estimation at bus bays: A probabilistic approach. Transportation Research Part C: Emerging Technologies 36, 61–71.

Milkovits, M.N., 2008. Modeling the factors affecting bus stop dwell time: Use of automatic passenger counting, automatic fare counting, and automatic vehicle location data. Journal of Transportation Research Board 2072(1), 125–130.

Nayeem, M.A., Rahman, M.K., Rahman, M.S., 2014. Transit network design by genetic algorithm with elitism. Transportation Research Part C: Emerging Technologies 46, 30–45.

Petersen, H.L., Larsen, A., Madsen, O.B.G., Petersen, B., Ropke, S., 2012. The simultaneous vehicle scheduling and passenger service problem. Transportation Science 47(4), 603–616.

Pretty, R.L., Russell, D.J., 1988. Bus boarding rates. Australian Road Research 18(3).

Rajbhandari, R., Chien, S.I., Daniel, J.R., 2003. Estimation of bus dwell times with automatic passenger counter information. Transportation Research Record: Journal of the Transportation Research Board 1841, 120–127.

Rexfelt, O., Schelenz, T., Karlsson, M., Suescun, A., 2014. Evaluating the effects of bus design on passenger flow: Is agent-based simulation a feasible approach? Transportation Research Part C: Emerging Technologies 38, 16–27.

Shalaby, A., Farhan, A., 2004. Prediction model of bus arrival and departure times using avl and apc data. Journal of Public Transportation 7(1), 41–62.

Strathman, J.G., Dueker, K.J., Kimpel, T., 2000. Service reliability impacts of computer-aided dispatching and automatic vehicle location technology: A tri-met case study. Transportation Quarterly 54(3), 85–102.

Sun, A., Hickman, M., 2008. The holding problem at multiple holding stations. Computer-aided Systems in Public Transport 600, 339–359.

Szeto, W., Jiang, Y., Wong, K., Solayappan, M., 2011. Reliability-based stochastic transit assignment with capacity constraints: Formulation and solution method. Transportation Research Part C: Emerging Technologies 35, 286–304.

Tan, J., Li, Z., Li, L., Zhang, Y., Lu, L., 2014. Berth assignment planning for multi-line bus stops. Journal of Advanced Transportation 48(7), 750–765.

Tirachini, A., 2013. Bus dwell time: the effect of different fare collection systems, bus floor level and age of passengers. Transportmetrica A: Transport Science 9(1), 28–49.

Tirachini, A., Hensher, D.A., 2011. Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. Transportation Research Part B: Methodological 45(5), 828–844.

TRB, 2013. Highway Capacity Manual. Transportation Research Board. Washington, DC.

Tyler, N., Silva, P., Brown, N., Fernndez, R., 2002. Operational impacts of bus stops. Accessibility and the bus system: From concepts to practice,. Thomas Telford, London.

Vuchic, V.R., 2005. Urban transit: operations, planning, and economics. Wiley, Hoboken, NJ.

Wu, Y., Tang, J., Yu, Y., Pan, Z., 2015. A stochastic optimization model for transit network timetable design to mitigate the randomness of traveling time by adding slack time. Transportation Research Part C: Emerging Technologies 52, 15–31.

Yao, B., Hu, P., Lu, X., Gao, J., Zhang, M., 2014. Transit network design based on travel time reliability. Transportation Research Part C: Emerging Technologies 43, 233–248.

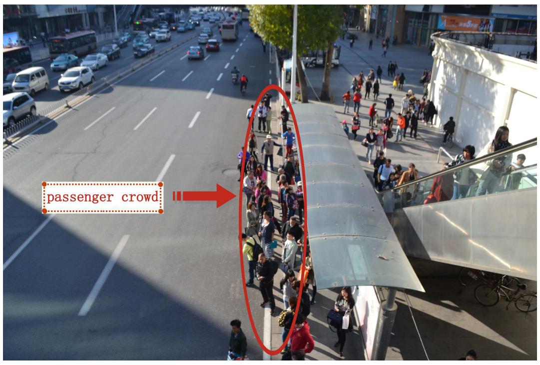
Yu, B., Lam, W.H., Tam, M.L., 2011. Bus arrival time prediction at bus stop with multiple routes. Transportation Research Part C: Emerging Technologies 19(6), 1157–1170.

Yu, B., Yang, Z.Z., Jin, P.H., Wu, S.H., Yao, B.Z., 2012. Transit route network design-maximizing direct and transfer demand density. Transportation Research Part C: Emerging Technologies 22, 58–75.

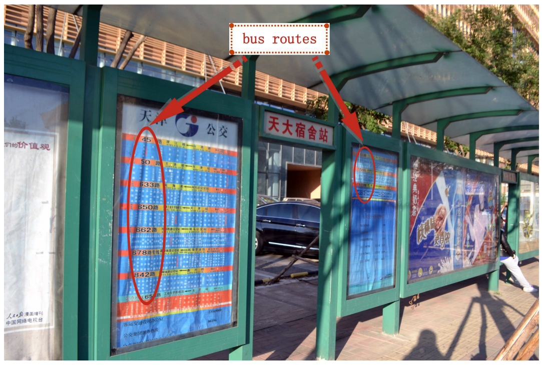
Zhang, L., Yang, H., Wu, D., Wang, D., 2014. Solving a discrete multimodal transportation network design problem. Transportation Research Part C: Emerging Technologies 49, 73–96.

List of Figures

1 Phenomena associated with bus stop areas. 24
2 The structure of the curbside bus stop. 25
3 Time-space diagram of service time estimation model. 26
4 The four possible scenarios occurring at a curbside bus stop with the two berths. 27
5 Variation of T_s VS. Number of iteration. 28
6 The four possible scenarios with allowable overtaking. 29
7 The impact of overtaking. 30



(a) Disorder passengers in a curbside bus stop.



(b) Bus route postings at a bus stop.

(c) Queuing phenomenon at a bus stop area.

(d) Limited number of berths.

Figure 1: Phenomena associated with bus stop areas.

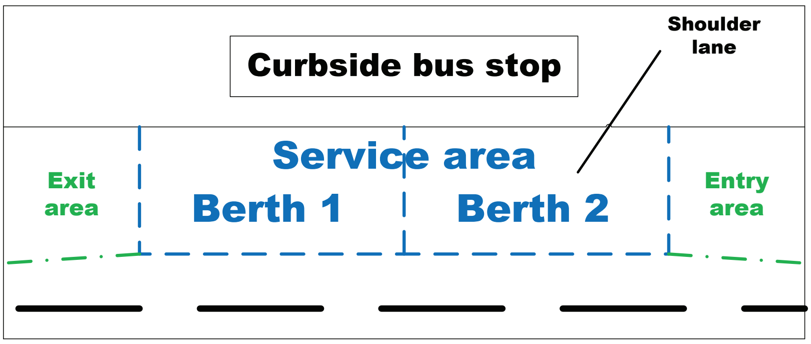


Figure 2: The structure of the curbside bus stop.

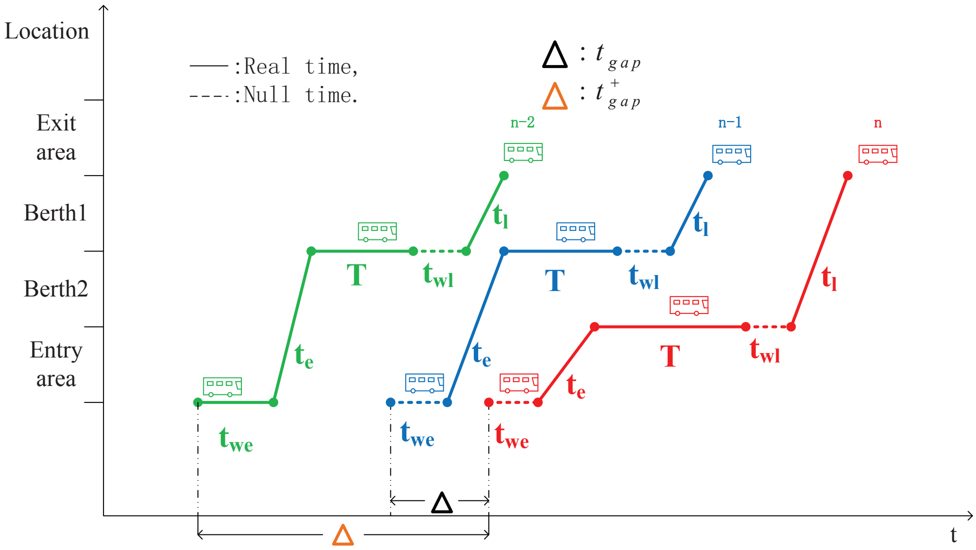
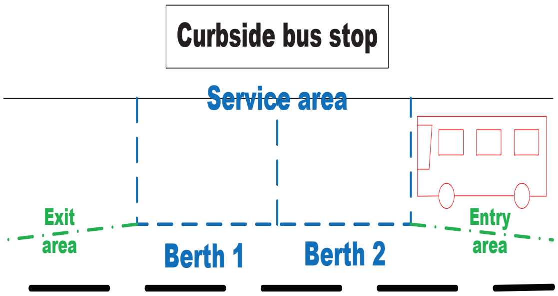
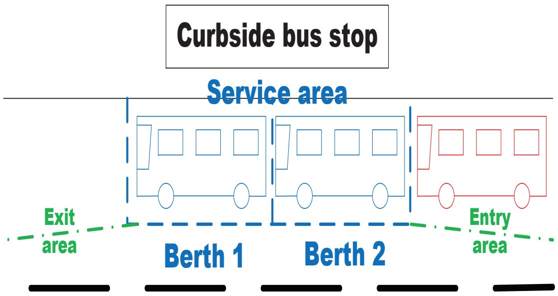


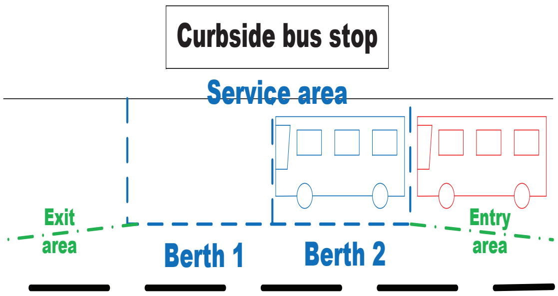
Figure 3: Time-space diagram of service time estimation model.



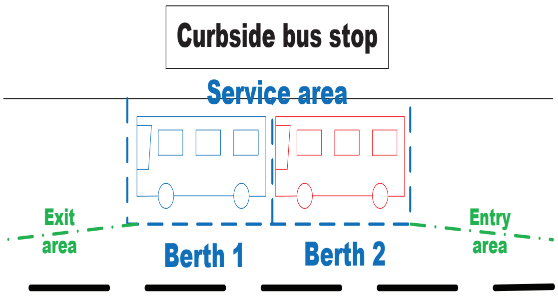
(a) Scenario A.



(b) Scenario B.



(c) Scenario C.



(d) Scenario D.

Figure 4: The four possible scenarios occurring at a curbside bus stop with the two berths.

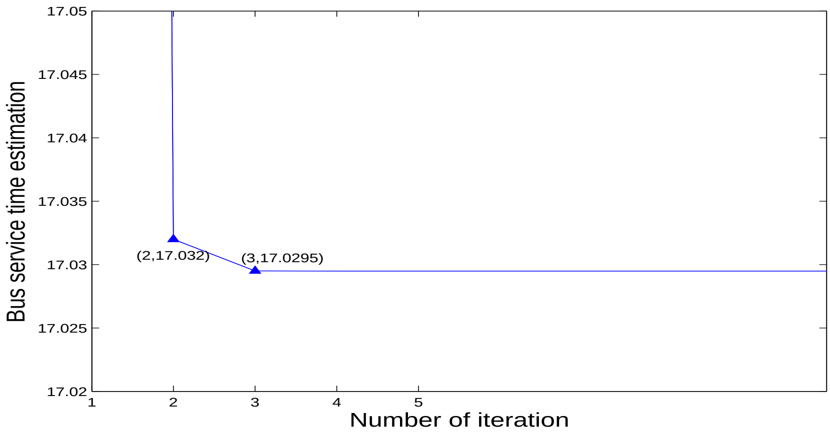
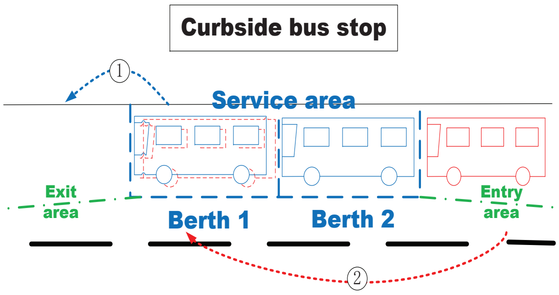
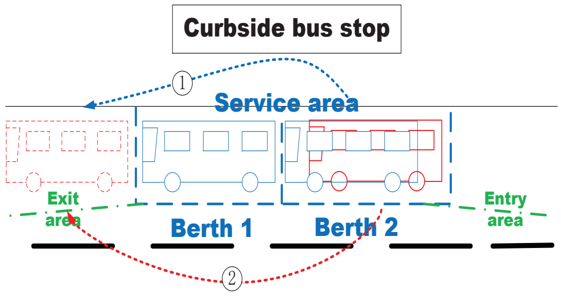


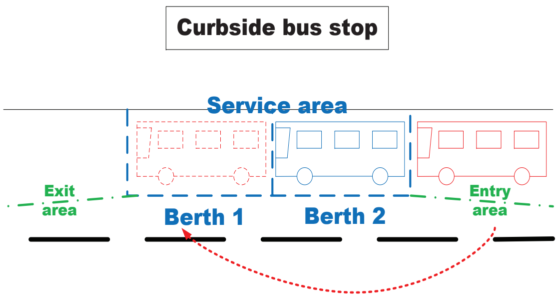
Figure 5: Variation of T_s VS. Number of iteration.



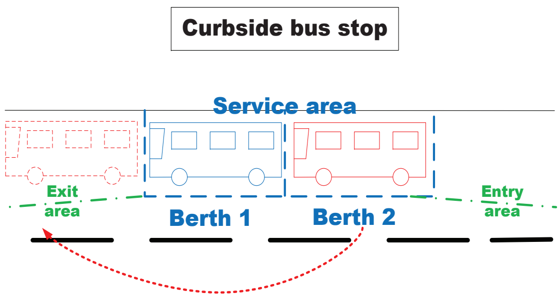
(a) Scenario F-1.



(b) Scenario F-2.



(c) Scenario G.



(d) Scenario H.

Figure 6: The four possible scenarios with allowable overtaking.

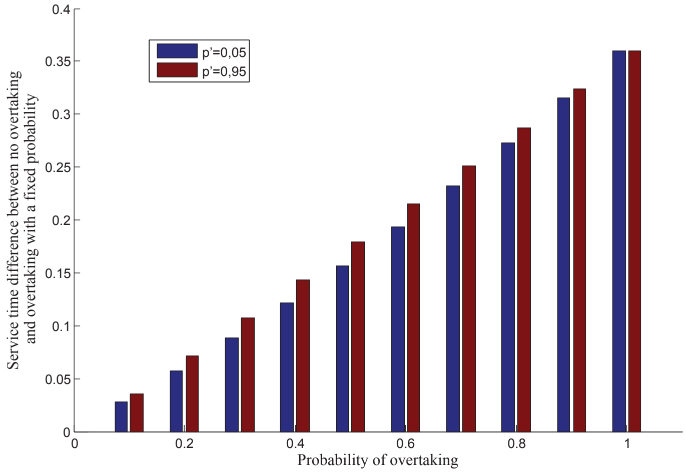


Figure 7: The impact of overtaking.

List of Tables

1 Variables and parameters 16
2 Sample data 32
3 Results of bus arrival rate 33
4 Values of constants 34
5 Results of bus service time estimation 35
6 Error measurements of CPSTM 36
7 Error measurements of basic model 37
8 Differences of error measurements of basic model and CPSTM 38
9 Error measurements of CPSTM with observed t_{gap}, t_{gap}^{+} and t_{gap}^{j} 39
10 Differences of error measurements of CPSTM and CPSTM with observed data 40
11 Error measurements of possion v.s. non-possion 41

Table 2: Sample data

t_{we}	t_e	n	$\sum^{n} a_h$	m	$\sum^{m} b_q$	C	T	t_{wl}	t_l	T_s	t_{gap}	t_{gap}^{+}
8	4	4	14	1	1	3	17	0	3	32	5	18
7	11	2	6	0	0	5	11	0	4	33	7	11
0	4	3	21	3	4	1	22	0	5	31	26	126

Table 3: Results of bus arrival rate

Bus line number(No.j)	Arrival rate λ_j(buses/hour)
No. 45	3.39
No. 50	6.48
No. 633	6.87
No. 662	6.10
No. 678	5.03
No. 842	8.03
No. 851	4.16
No. 859	5.23
No. 879	4.16
Total bus arrival rate $\lambda_0 = 49.45$	

Table 4: Values of constants

Content	Value
deadtime C of No. 45	6.94s
deadtime C of No. 50	5.13s
deadtime C of No. 633	2.51s
deadtime C of No. 662	4.29s
deadtime C of No. 678	3.11s
deadtime C of No. 842	4.39s
deadtime C of No. 851	4.39s
deadtime C of No. 859	5.45s
deadtime C of No. 879	4.96s
bus length L	12m
bus speed v_e	10.54km/h
bus speed v_l	9.6km/h
alighting time of per passenger y_a	1.24s
boarding time of per passenger y_b	2.26s

Table 5: Results of bus service time estimation

Bus line number(No. j)	Estimation of bus service time(s)
No. 45	18.34
No. 50	18.53
No. 633	12.80
No. 662	16.64
No. 678	13.57
No. 842	15.41
No. 851	21.15
No. 859	16.84
No. 879	22.37
Estimation of $T_s = 17.03$	

Table 6: Error measurements of CPSTM

Bus line number (No.j)	ε^j_{mean}	μ^j_{mean}	ε^j_{rmean}
No. 45	5.03	6.32	24.5%
No. 50	4.85	6.73	25.2%
No. 633	2.49	3.22	18.5%
No. 662	3.90	4.85	26.0%
No. 678	1.99	2.62	15.1%
No. 842	4.11	5.65	22.2%
No. 851	5.82	6.79	32.1%
No. 859	4.08	5.98	18.6%
No. 879	5.48	6.81	27.4%
Total mean absolute error $\overline{\varepsilon_{mean}} = 4.15$(s)			
Total root mean squared error $\overline{\mu_{mean}} = 5.62$(s)			
Total mean relative error $\overline{\varepsilon_{rmean}} = 23.8\%$			

Table 7: Error measurements of basic model

Bus line number (No.j)	ε'^{j}_{mean}	μ'^{j}_{mean}	ε'^{j}_{rmean}
No. 45	9.47	9.68	49.1%
No. 50	8.13	8.19	42.7%
No. 633	8.22	8.31	63.5%
No. 662	7.77	7.93	49.1%
No. 678	7.44	7.47	57.9%
No. 842	9.39	9.80	53.2%
No. 851	7.74	7.90	41.9%
No. 859	9.31	9.40	51.8%
No. 879	10.11	10.81	47.8%
Total mean relative error $\overline{\varepsilon'_{mean}} = 8.63(s)$			
Total root mean squared error $\overline{\mu'_{mean}} = 8.91(s)$			
Total mean relative error $\overline{\varepsilon'_{rmean}} = 51.1\%$			

Table 8: Differences of error measurements of basic model and CPSTM

Bus line number (No.j)	$\varepsilon^{'j}_{mean} - \varepsilon^{j}_{mean}(s)$	$\mu^{'j}_{mean} - \mu^{j}_{mean}$(s)	$\varepsilon^{'j}_{rmean} - \varepsilon^{j}_{rmean}$
No. 45	4.44	3.36	24.6%
No. 50	3.28	1.46	17.5%
No. 633	5.73	5.09	45.0%
No. 662	3.87	3.08	23.1%
No. 678	5.45	4.85	42.8%
No. 842	5.28	4.15	31.0%
No. 851	1.92	1.11	9.8%
No. 859	5.23	3.42	33.2%
No. 879	4.63	4.00	20.4%
$\overline{\varepsilon^{'}_{mean}} - \overline{\varepsilon_{mean}} = 4.48(s)$			
$\overline{\mu^{'}_{mean}} - \overline{\mu_{mean}} = 3.29(s)$			
$\overline{\varepsilon^{'}_{rmean}} - \overline{\varepsilon_{rmean}} = 27.3\%$			

Table 9: Error measurements of CPSTM with observed t_{gap}, t_{gap}^{+} and t_{gap}^{j}

Bus line number (No.j)	ε''^{j}_{mean}	μ''^{j}_{mean}	$\varepsilon''^{j}_{rmean}$
No. 45	4.79	6.02	22.6%
No. 50	4.77	6.47	23.3%
No. 633	2.47	3.25	18.2%
No. 662	4.09	5.41	26.8%
No. 678	2.14	2.74	16.6%
No. 842	3.91	5.64	21.2%
No. 851	5.92	7.33	32.2%
No. 859	3.79	5.69	17.2%
No. 879	6.00	7.61	27.5%
Total mean absolute error $\overline{\varepsilon_{mean}''} = 4.16$(s)			
Total root mean squared error $\overline{\mu_{mean}''} = 5.76$(s)			
Total mean relative error $\overline{\varepsilon_{rmean}''} = 23.2\%$			

Table 10: Differences of error measurements of CPSTM and CPSTM with observed data

Bus line number No.j	$\varepsilon^{j}_{mean} - \varepsilon''^{j}_{mean}(s)$	$\mu^{j}_{mean} - \mu''^{j}_{mean}$(s)	$\varepsilon^{j}_{rmean} - \varepsilon''^{j}_{rmean}$
No. 45	0.24	0.30	1.9%
No. 50	0.08	0.26	1.9%
No. 633	0.02	-0.03	0.3%
No. 662	-0.19	-0.56	-0.8%
No. 678	-0.15	-0.12	-1.5%
No. 842	0.2	0.01	1.0%
No. 851	-0.10	-0.54	-0.1%
No. 859	0.29	0.29	1.4%
No. 879	-0.52	-0.8	-0.1%
$\overline{\varepsilon_{mean}} - \overline{\varepsilon''_{mean}} = -0.01(s)$			
$\overline{\mu_{mean}} - \overline{\mu''_{mean}} = -0.14(s)$			
$\overline{\varepsilon_{rmean}} - \overline{\varepsilon''_{rmean}} = 0.6\%$			

Table 11: Error measurements of possion v.s. non-possion

	ε^j_{mean}			μ^j_{mean}			ε^j_{rmean}		
No.j	possion	uniform	normal	possion	uniform	normal	possion	uniform	normal
No.45	5.03	4.99	4.89	6.32	6.25	6.10	24.5%	24.4%	24.5%
No.50	4.85	4.86	4.87	6.73	6.60	6.60	25.2%	26.8%	26.8%
No.633	2.49	2.51	5.55	3.22	3.17	6.17	18.5%	19.3%	48.2%
No.662	3.90	3.97	4.05	4.85	4.88	4.93	26.0%	27.2%	28.1%
No.678	1.99	2.00	2.22	2.62	2.62	2.69	15.1%	15.2%	17.5%
No.842	4.11	3.99	3.86	5.65	5.50	5.30	22.2%	21.9%	21.9%
No.851	5.82	6.15	6.23	6.79	7.24	7.31	32.1%	35.0%	40.0%
No.859	4.08	4.08	4.14	5.98	5.86	5.63	18.6%	18.6%	20.4%
No.879	5.48	5.47	5.73	6.81	6.80	6.92	27.4%	27.3%	29.8%